

Electrons as Quasi-Bosons in Strong Magnetic Fields and the Stability of Magnetars

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Following the recent ideas of Dryzek, Kato, Muñoz and Singleton [1] (henceforth we call this paper as R1), that the composite system consisting of an electron along with its screened electric field and within the sphere of influence, the trapped magnetic field of white dwarf can behave like a boson, we have argued that such an exotic transition (bosonization) of electronic component in strongly magnetized neutron star matter in β -equilibrium can make the existence of magnetars physically viable.

One of the oldest subject of physics, "the study of the effect of strong magnetic field on dense matter of charged particles" has gotten a new dimension after the discovery of a few magnetars. These exotic objects are believed to be strongly magnetized young neutron stars of surface magnetic field $\approx 10^{15}\text{G}$ and at the core region it may go up to 10^{18}G [2–6]. It is therefore very much advisable to examine thoroughly the effect of such strong magnetic field on various physical properties of dense neutron star matter as well as on the physical processes taking place inside neutron stars. An extensive studies have already been done on the equation of state of dense neutron star matter in presence of strong magnetic field. Such studies are based on the quantum mechanical effect of strong magnetic field [7,8]. The effect of strong quantizing magnetic field on the gross properties, e.g., the mass, radius, moment of inertia etc., of neutron stars, which are strongly dependent on the equation of state of matter have been obtained [9,10]. How the weak processes, e.g., weak reactions and decays are affected by the quantum mechanical effect of strong magnetic field have also been derived from the first principal [11]. These studies show that the β -equilibrium condition too depends on the strength of magnetic field. Since the cooling of neutron stars is dominated by the emission of neutrinos produced by weak processes inside the stars, these studies also give an idea of the effect of strong magnetic field on the thermal evolution of neutron stars. Not only that, the presence of strong magnetic field can change significantly, both qualitatively and quantitatively, the transport coefficients (e.g., viscosity coefficient, thermal conductivity, electrical conductivity etc.) of dense neutron star matter [12–14]. The magnetic field can change the tensorial character of transport coefficients of neutron star matter. Such qualitative changes in transport coefficients can cause significant changes in thermal evolution of neutron star matter and also the evolution of its magnetic field. There are another kind of studies; the effect of strong quantizing magnetic field on quark-hadron phase transition. It was shown explicitly that a first order quark-hadron phase transition is absolutely forbidden if the strength of magnetic field exceeds 10^{15}G . However, a metal insulator type (color insulator to color metal) second order phase transition is possible unless the field strength exceeds 10^{20}G [15,16]. It has also been shown, that even if there is a first order quark-hadron phase transition for magnetic field strength $< 10^{15}\text{G}$ at the core region of a neutron star, an investigation of chemical evolution of quark matter, with various initial conditions, leads to the system in β -equilibrium, revealed that the system becomes energetically unstable in chemical equilibrium [17]. In a class of completely different type of studies, the mechanical stability and some of the gross properties of deformed stellar objects are analyzed with general relativity. The presence of strong magnetic field can destroy the spherical symmetry of a neutron star [18–20]. Then it is quite possible for a deformed and rotating neutron star to emit gravity waves, which in principle can be detected [21]. It has also been shown with the help of general relativity that the presence of strong magnetic field may pose a serious problem on the stability of magnetars, in the extreme case it may become either a black disk or black strings [22]. In a very recent work, we have critically studied the ferro-magnetism of neutron star matter, which is believed to be one of the sources of residual magnetism of old neutron stars/sources of magnetic field of millisecond pulsars. In these studies, we have shown explicitly that spontaneous ferromagnetic transition in absence of an external magnetic field is absolutely forbidden in neutron star matter in β -equilibrium [23]. However in the case of neutrino trapped neutron star matter (proto-neutron star matter), the possibility of such a transition can not be ruled out, provided the neutrinos carry some non-zero mass. We have also analyzed the problem of occupancy of only zeroth Landau levels by electrons/protons, which occur in presence of ultra-strong

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magnetic fields [24]. It has been argued in this microscopic model analysis that in presence of a strong quantizing magnetic field the existence of neutron star matter in β -equilibrium is questionable. Which further opens up a vital question on the possibility of magnetars as young and strongly magnetized neutron stars. The macroscopic general relativistic studies and the microscopic model calculations therefore arrive at the same conclusion- "the nonexistence of magnetars". To resolve this controversy, we have followed the ideas of reference R1. We have also assumed that it is possible for the electrons to become quasi-bosons after combining with the screened electric field of the electron and the magnetic field of the star trapped within the screened region. Since pions or kaons can not condense in presence of ultra strong magnetic field [25], it is therefore not energetically favorable for the system to produce π^- or K^- instead of electrons. On the other hand, the condensation of quasi-bosonic states of composite electron system to almost zero kinetic energy state is energetically more favorable even with respect to the occupancy of only the zeroth Landau level for electrons. Now it is found in the literature that fermi-bose transition plays a significant role in various branches of physics, e.g., high energy particle physics, and condensed matter physics. We therefore consider the feasibility of bosonization of electrons inside the magnetars to solve this serious problem of instability of neutron star matter in β -equilibrium in presence of ultra strong magnetic field and hence try to resolve the controversial issue of the existence of magnetars.

In the recent work (R1), the possibility of bosonization of electrons in magnetized white dwarfs has been proposed. In this paper, some of the interesting properties of white dwarfs relevant for such exotic transition have been discussed. The collapse of white dwarfs to neutron stars, because of bosonization has also been addressed in this reference.

Now several authors have investigated the fermi-bose transition during the last few decades in various branches of physics, e.g., in high energy particle physics, condensed matter physics - especially in strongly correlated electron system, fractional quantum Hall effect, anyon physics etc.

In the case of high energy particle physics, the application of fermi-bose transition has been studied in both abelian as well as in non-abelian field theories and also in non-linear Sine-Gordon model, in the low dimension and later, extended to $3 + 1$ dimensions [26–28]. In the condensed matter physics, there are certain systems in which fermions can be converted to quasi-bosons. The bosonization has been studied in detail for the crystalline as well as for the disordered condensed matter systems. Such studies are mainly concentrated in the low dimensional cases. However, an extension to $3 + 1$ dimensions have also been done for certain cases. In $2D$, the anyons, which are the composite system of fermions and quantized magnetic flux, satisfy any statistics, have also been studied. The other interesting studies are on the strongly correlated electron system, the mesoscopic systems and the quantum Hall effect, and the high T_c super-conductivity [29–33].

The theoretical model presented in reference (R1) is the first attempt to apply bosonization in the study of compact stellar objects. Following the ideas proposed in this reference, we shall try to convince that if electrons in magnetars are converted to quasi-bosons, the serious problem on the existence of magnetars and also on the stability of dense neutron star matter in β -equilibrium in presence of extremely strong magnetic field posed by the electrons occupying only the zeroth Landau levels can in principle be solved.

We assume that the electrons in magnetars can behave like quasi-bosons in combination with its screened coulomb field and the strong magnetic field trapped within the sphere of influence. The size of these composite objects is $\sim R_{sc} \sim 10^{-11} - 10^{-12}$ cm in the relativistic region, where R_{sc} is the screening length of the electrostatic field of electrons. To obtain the screening length we use the relativistic version of Thomas-Fermi model. According to which, the chemical potential of the electron is given by

$$\mu_e = (p_{F_e}^2(r) + m_e^2)^{1/2} - e\phi(r) = \text{constant} \quad (1)$$

where $p_{F_e}(r)$, m_e and e are respectively the fermi momentum, mass and magnitude of charge of the electron and ϕ is the effective electrostatic field experienced by the electron. The well known Poisson's equation is then given by

$$\nabla^2\phi = -4\pi en'_p(r) + 4\pi en'_e(r) \quad (2)$$

where n'_p and n'_e are respectively the perturbed densities of proton and electron from their equilibrium values n_p^0 and n_e^0 satisfy the relation $n_p^0 = n_e^0$, to fulfill the charge neutrality condition. Then it is quite obvious that in the perturbed case, the total electron density is given by

$$n_e = n_e^0 + n'_e \quad (3)$$

Now

$$n_e = \frac{1}{3\pi^2} p_{F_e}^3 = \frac{1}{3\pi^2} \{(\mu_e + e\phi)^2 - m_e^2\}^{3/2} \quad (4)$$

Hence we get for small ϕ approximation

$$n'_e(r) \approx \frac{3en_e^0\phi(r)\mu_e}{\mu_e^2 - m_e^2} \quad (5)$$

On substituting n'_e in the Poisson's equation, we have the screening length

$$R_{sc} = \left(\frac{\mu_e^2 - m_e^2}{12\pi e^2 n_e^0 \mu_e} \right)^{1/2} \quad (6)$$

Now the angular momentum of electromagnetic field (combination of screened electrostatic field of electron and the strong magnetic field of the magnetar trapped within this region) is given by (we have assumed $\hbar = c = 1$)

$$\vec{L}^{em} = 2eBR_{sc}^2 \hat{z} \quad (7)$$

where \hat{z} is an unit vector along Z-axis which is also assumed to be the direction of strong magnetic field B . Then the total spin of the composite system is given by

$$\vec{S}_{eff} = \vec{S}_e + \vec{L}^{em} \quad (8)$$

If the eigen value of L^{em} is an odd integer multiple of $1/2$, the composite system behaves like a quasi-boson.

To explain how the bosonization of electron gas depends on matter density and to obtain the corresponding critical value of magnetic field strength at which the composite system behaves like a quasi-boson, we put

$$L^{em} = \frac{s}{2} \quad (9)$$

where s is a positive odd integer parameter. The quasi-bosons will therefore behave either like scalar, vector or tensor particles depending on the value of s .

In fig.1 we have plotted the critical strength of magnetic field against the baryon number density for various values of the parameter s . This figure shows that if there is possibility of any such composite objects of electrons along with its screened electro-static field and the magnetic field of the star trapped within the sphere of radius R_{sc} , then the quasi-bosons are not necessarily of scalar or vector type, but they can have tensorial character too of rank ≥ 2 , depending on the magnitude of magnetic field strength. Although, theoretically predicted upper limit for spin of boson is 2, which is called graviton, it is in principle possible to have quasi-bosons (which are of course not real bosons) of spin ≥ 2 inside the magnetars, especially, if the magnetic field at the core region $\sim 10^{18}\text{G}$, for which the maximum value of spin for quasi-boson states at the core region can be $4\hbar$. Since these composite objects are not real bosons and also have finite dimension ($\sim R_{sc}$), then instead of comparing them with real bosons, it is better to draw some analogy with the high spin states of nuclei formed in nuclear fusion reactions. The finite dimension of these composite objects justifies to draw the comparison with high spin states of the nuclei. Whatever be the corresponding real world picture, these quasi-bosons should condense within the magnetar. Since the stability of the neutron stars is governed by the degeneracy pressure of neutron matter, the condensation (transition to almost zero momentum state) of the quasi-bosons does not affect the mechanical equilibrium of the system. Further, the problem arises out of the occupancy of only the zeroth Landau level by electrons is also solved. As a consequence, the stability of strongly magnetized dense neutron star matter in β -equilibrium is gurented. Now there are two different kinds of general relativistic studies on the stability of strongly magnetized neutron stars. In purely classical studies, it has been shown that the inward magnetic pressure along the magnetic axis deforms the spherically symmetric structure of a star to an oblate shape. In the extreme case, it can become a black disk. Which of course needs a very high magnetic field ($\geq 10^{20}\text{G}$). If it is so, the instability of strongly magnetized neutron stars as predicted by classical general relativity can not be solved with microscopic (bosonization) model described in this article. In the other type of general relativistic studies, the effect of paramagnetism of Landau diamagnetic system has been considered. Now the paramagnetic behavior of Landau diamagnetic system dominates if only the zeroth Landau levels are occupied by the charged fermions. In this case all the spins are aligned. In such a scenario, the spherically symmetric structure of the star will be distorted to prolate shape and in the extreme case, it takes the shape of a cigar or reduces to

a black string (one dimensional object). Since this kind of instability is connected to the paramagnetic behavior of charged fermions (mainly electrons) occupying Landau levels, the possibility of bosonization of electronic components can solve this problem. Which therefore removes the controversy on the non-existence of magnetars.

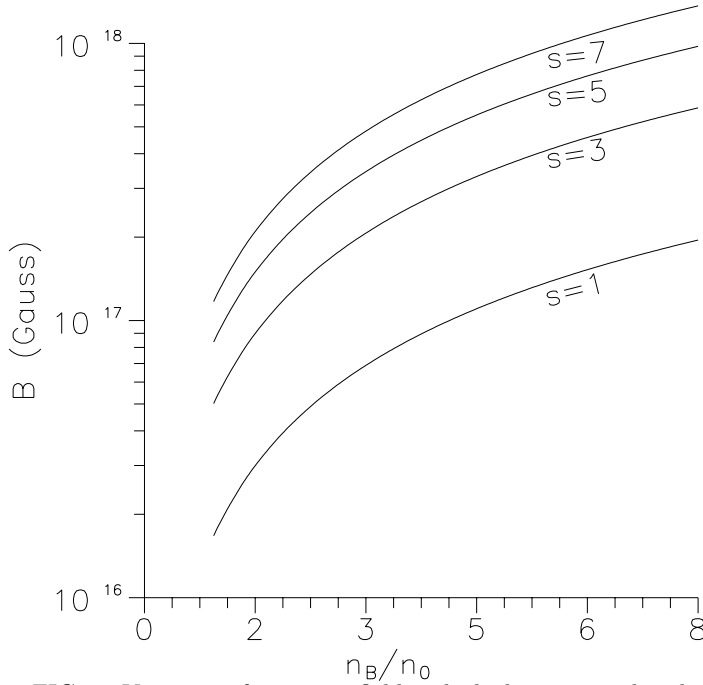


FIG. 1. Variation of magnetic field with the baryon number density (expressed in terms of normal nuclear density) for various values of the parameter s .

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